2-D Direction of Arrival Estimation Combining UCA-RARE and MUSIC for Uniform Circular Arrays Subject to Mutual Coupling

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***Abstract* — A new algorithm for 2-D direction-of-arrival estimation of azimuth and elevation angle is presented for uniform circular arrays by combining UCA-RARE and MUSIC in the presence of the mutual coupling. To describe mutual coupling in a uniform circular array we make use of the symmetry of the array and expand the open-circuit voltages into a limited number of spherical modes. The UCA-RARE technique is then applied to estimate the azimuth angle independent from the elevation angle. Next for each azimuth angle we estimate the corresponding elevation angles. Some illustrative examples are given to validate our approach.**

1 INTRODUCTION

Estimating the directions of arrival (DOAs) of plane waves impinging on antenna arrays is an important issue in mobile communication systems. In recent years several algorithms were proposed to estimate DOAs [1]. The performance of these algorithms is however often affected by mutual coupling between the antenna elements in the array. Especially in uniform circular arrays (UCAs) mutual coupling can be significant. In [2] it is proven that the open-circuit voltage of each element in the UCA can be described with a limited number of parameters by means of a spherical mode expansion. Another challenge is the simultaneous estimation of azimuth and elevation angles (φ, θ). UCA-ESPRIT [1] is a useful technique for 2-D angle estimation, but the difficulty to compensate for mutual coupling is a disadvantage. Other methods which are compatible with mutual coupling effects are mostly restricted to a one-dimensional search over φ which is numerically easy to implement. The extension to 2D angle estimation is computationally expensive. The aim is to develop a method which overcomes these problems.

In this paper we combine two algorithms, being UCA-RARE [3] and MUSIC [4]. In [3] a new eigenstructure-based estimation method (UCA-RARE) is developed and first of all we combine this algorithm with a phase-mode expansion for the mutual coupling effects, allowing to estimate the azimuth parameters of the source directions φ decoupled from the elevation angles in the presence of mutual coupling. Given the knowledge of the azimuth angles, we estimate the elevation parameters by performing a one-dimensional search over the MUSIC-spectrum. For constructing the MUSIC-spectrum we rely on the spherical mode expansion that describes mutual coupling in the uniform circular array. In section 2 the combination of the two search algorithms is explained. The new formalism includes the full electromagnetic behaviour of the uniform circular array. In section 3 some illustrative results are presented, proving that the simultaneous estimation of azimuth and elevation parameters is realized by combining the proposed algorithms.

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2 THEORY

2.1 UCA-RARE in presence of mutual coupling

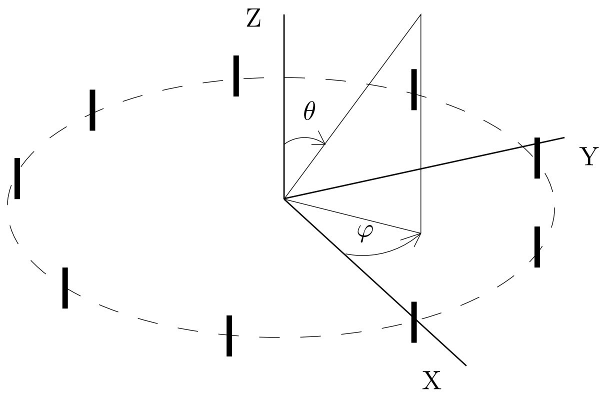

Figure 1: uniform circular array composed of nine antenna elements.

Consider a uniform circular array (UCA) consisting of Z antenna elements. The antennas are distributed uniformly over a circle with radius R. The antenna elements operate at a frequency with a corresponding wave number k. The phase center of each antenna element is located in the xy-plane, at azimuth angles $\varphi_t = (t-1)\frac{2\pi}{Z}$ with $t = 1 \ldots Z$.

We assume that all antennas are identical. Due to mutual coupling between the different antenna elements the directional patterns of the antennas are different from the directional pattern of the standalone elements. Symmetry of the array, however, results in the following relationship between the open-circuit voltages of different antenna elements:

$$V_{0,t}(\varphi,\theta) = V_{0,1}(\varphi - \varphi_t, \theta). \tag{1}$$

As UCA-RARE only relies on this symmetry property and not on the detailed knowledge of the antenna patterns, it can be used in array configurations in which mutual coupling effects are significant. Let q narrow-band source signals impinge on the array from the unknown DOAs $\{\varphi_l, \theta_l\}_{l=1}^{q}$, where $\varphi_l \in [0, 2\pi[$ is the azimuth angle of the lth source and $\theta_l \in [0,\pi]$ is the corresponding elevation angle (see figure 1). The data model is given by

$$\mathbf{x}(t) = \overline{\overline{A}}(\varphi,\theta)\mathbf{s}(t) + \mathbf{n}(t). \tag{2}$$

Here $\mathbf{s}(t) \in \mathbb{C}^{q\times 1}$ is the signal vector, $\mathbf{n}(t) \in \mathbb{C}^{Z\times 1}$ is additive white Gaussian noise, and the $(Z \times q)$ element space manifold matrix

$$\overline{\overline{A}}(\varphi,\theta) = [\mathbf{a}(\varphi_1,\theta_1) \quad \mathbf{a}(\varphi_2,\theta_2)\ldots\mathbf{a}(\varphi_q,\theta_q)] \tag{3}$$

is composed of the array manifold $\{\mathbf{a}(\varphi_l,\theta_l)\}_{l=1}^{q}$ with the kth entry defined as $[\mathbf{a}(\varphi_l,\theta_l)]_k = V_{0,1}(\varphi_l - \varphi_k, \theta_l)$. In [5] it is proven that in a UCA, in the presence of mutual coupling, the open-circuit voltages can be expanded in a limited numbers of phase modes

$$a_k(\varphi,\theta) = a_1(\varphi - \varphi_k, \theta) = \sum_{m=-M}^{+M} a_m(\theta) e^{jm(\varphi-\varphi_k)}, \tag{4}$$

where M depends on the dimensions of the antenna array

$$M \gg kR. \tag{5}$$

After a beamspace transformation the data model becomes

$$\mathbf{x}_{beam}(t) = \overline{\overline{W}}\mathbf{x}(t) \tag{6}$$

$$[W]s,t = \frac{1}{\sqrt{Z}} e^{j\frac{2\pi}{Z}(s-M-1)(t-1)} \tag{7}$$

with $t = 1\ldots Z$ and $s = 1\ldots 2M+1$. For DOA estimation we calculate the beamspace data covariance matrix and the eigendecomposition is

$$\begin{aligned}\widehat{R} &= E\{\mathbf{x}_{beam}(t)\mathbf{x}_{beam}(t)^H\}\\ &= \widehat{E}_S\widehat{\Lambda}_S\widehat{E}_S^H + \widehat{E}_N\widehat{\Lambda}_N\widehat{E}_N^H\end{aligned}$$

The diagonal matrices $\widehat{\Lambda}_N \in \mathbb{R}^{(2M+1-q)\times(2M+1-q)}$ and $\widehat{\Lambda}_S \in \mathbb{R}^{q\times q}$ contain the noise-subspace and signal-subspace eigenvalues of $\widehat{R}$, respectively. The columns of the matrices $\widehat{E}_N \in \mathbb{C}^{(2M+1)\times(2M+1-q)}$ and $\widehat{E}_S \in \mathbb{C}^{(2M+1)\times q}$ denote the corresponding noise-subspace and signal-subspace eigenvectors. The beamspace MUSIC algorithm estimates the signal DOAs from the q deepest nulls of the MUSIC function

$$\widehat{f}_{MUSIC}(\theta,\varphi) = \mathbf{a}_{beam}^H(\theta,\varphi)\widehat{E}_N\widehat{E}_N^H\mathbf{a}_{beam}(\theta,\varphi) = 0 \tag{8}$$

with $\mathbf{a}_{beam}(\varphi,\theta) = \overline{\overline{W}}\mathbf{a}(\varphi,\theta)$. The beamspace manifold vector can be rewritten as

$$\mathbf{a}_{beam} = \overline{\overline{T}}(z)\mathbf{g}(\theta) \tag{9}$$

$$[\mathbf{g}(\theta)]_k = a_{M-k+1}(\theta) \tag{10}$$

$$\overline{\overline{T}}(z) = \begin{bmatrix} \overline{\overline{Q}}(z) & 0 \\ 0 & 1 \\ \Pi\overline{\overline{Q}}(1/z) & 0 \end{bmatrix} \tag{11}$$

$$\overline{\overline{Q}}(z) = diag\{z^{-M}, z^{-M+1}, \ldots, z^{-2}, z^{-1}\} \quad \text{with} \quad z = e^{j\varphi}, \tag{12}$$

where Π is the $M \times M$ anti-diagonal unit matrix. Notice that the part of the manofold vector, which contains the dependence of the elevation angle, consists of the different phase modes $a_m(\theta)$. In absence of mutual coupling these phase modes are replaced by the Bessel function of first kind [3]. Expression (9) is only valid when $Z > 2M$ and when the array manifold $\mathbf{a}(\varphi,\theta)$ can be described by a limited number of phase modes $\{-M\ldots+M\}$. The choice of M in (5) ensures that this condition is fulfilled. In the beamspace manifold vector we distinguish a part dependent on the elevation angle and a part dependent on the azimuth angle. This division between different angles is the key to the UCA-RARE algorithm where a larger set of the original manifold is studied:

$$\widehat{f}_{MUSIC}(\theta,\varphi) = \mathbf{g}^H(\theta)\overline{\overline{T}}^H(z)\widehat{E}_N\widehat{E}_N^H\overline{\overline{T}}(z)\mathbf{g}(\theta) = \tag{13}$$

$$c^H\overline{\overline{T}}(1/z)\widehat{E}_N\widehat{E}_N^H\overline{\overline{T}}(z)c = 0. \tag{14}$$

The nulls of (14) are found by the $1D$ polynomial criterion

$$P_{RARE}(z)_{|z|=1} = \det\{\overline{\overline{T}}(1/z)\widehat{E}_N\widehat{E}_N^H\overline{\overline{T}}(z)\} = 0. \tag{15}$$

The nulls give an estimation for $\{\varphi\}_{l=1}^{q}$ independent from the elevation angles, given the specific form (9) of the beamspace array manifold. The study of the larger set of the manifold implies that spurious states might be introduced. In [3] one proves that the spurious states are also solution of

$$P_{SPUR}(z)_{|z|=1} = \det\{\widehat{E}_S^H \, \overline{\overline{T}}(z) \, \Delta \, \overline{\overline{T}}(1/z) \, \widehat{E}_S\} \tag{16}$$

with $\Delta = (\overline{\overline{T}}(1/z)\overline{\overline{T}}(z))^{-1}$. Estimates of φ which are solution of both (15) and (16) are rejected.

2.2 MUSIC

In section 2.1 we have outlined the UCA-RARE approach to estimate the azimuth angles in the presence of mutual coupling. In order to determine the elevation angles we calculate the data covariance matrix and perform an eigendecomposition.

$$R = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \overline{\overline{E}}_S\overline{\overline{\Lambda}}_S\overline{\overline{E}}_S^H + \overline{\overline{E}}_N\overline{\overline{\Lambda}}_N\overline{\overline{E}}_N^H. \tag{17}$$

The diagonal matrices $\overline{\overline{\Lambda}}_N \in \mathbb{R}^{(Z-q)\times(Z-q)}$ and $\overline{\overline{\Lambda}}_S \in \mathbb{R}^{q\times q}$ contain the noise-subspace and signal-subspace eigenvalues of R, respectively. In turn, the columns of the matrices $\overline{\overline{E}}_N \in \mathbb{C}^{Z\times(Z-q)}$ and $\overline{\overline{E}}_S \in \mathbb{C}^{Z\times q}$ denote the corresponding noise-subspace and signal-subspace eigenvectors. The classical MUSIC function estimates DOAs by searching for the q deepest nulls in the (φ, θ) plane.

$$f_{MUSIC}(\varphi,\theta) = \mathbf{a}^H(\theta,\varphi)\overline{\overline{E}}_N\overline{\overline{E}}_N^H\mathbf{a}(\theta,\varphi) = 0 \tag{18}$$

The estimates of the azimuth angles from UCA-RARE make it possible to perform a 1D search over the MUSIC function, which is computationally more efficient than the 2D search. For every estimate of φ a 1D search is performed. In contrast to UCA-RARE we look for the highest peaks of the MUSIC spectrum.

$$g_{MUSIC}(\theta) = 1/f_{MUSIC}(\widehat{\varphi},\theta) \tag{19}$$

$$= 1/\mathbf{a}^H(\widehat{\varphi},\theta)\overline{\overline{E}}_N\overline{\overline{E}}_N^H\mathbf{a}(\widehat{\varphi},\theta). \tag{20}$$

For the calculation of $\mathbf{a}(\widehat{\varphi},\theta)$ we use the property that the array manifold of the antenna elements can be written as an expansion of a limited number of spherical modes [2]:

$$a_1(\varphi,\theta) = \sin\theta \sum_{m=-M}^{+M}\sum_{n=|m|}^{N} b_{mn}P_n^{|m|}(\cos\theta)e^{jm\varphi}, \tag{21}$$

with $M \gg kR$ and $N \gg k\sqrt{R^2 + z_{max}^2}$ (z_{max} is the largest z-value of the antenna element in the xyz-coordinate system). The mutual coupling in the antenna array is described in an rigorous manner by a limited number of parameters b_{mn}. This property allows us to calculate the array manifold $\mathbf{a}(\widehat{\varphi},\theta)$ for an arbitrary DOA (φ,θ) in the presence of mutual coupling. Practically $g_{MUSIC}(\theta)$ is computed over a discrete number of θ-values and a search for maxima is performed.

SNR	20dB	10dB	3dB
$\widehat{\varphi}_1$	50.0 (0.2)	50.0 (0.4)	50.0 (0.6)
$\widehat{\theta}_1$	49.8 (0.4)	49.8 (0.7)	49.7 (1.0)
$\widehat{\varphi}_2$	135.0 (0.4)	135.1 (0.8)	135.0 (1.4)
$\widehat{\theta}_2$	145.2 (0.4)	145.1 (0.8)	145.3 (1.3)
$\widehat{\varphi}_3$	280.0 (0.1)	280.0 (0.1)	280.0 (0.1)
$\widehat{\theta}_3$	89.8 (0.5)	89.5 (1.1)	88.9 (1.6)

Table 1: Mean and standard deviation (in degrees) of 500 implementations, for different SNR levels.

3 RESULTS

The algorithm to estimate DOAs in two dimensions is tested on a UCA of nine dipole antennas tuned to 900 MHz. The array elements are distributed uniformly over a circle with diameter $d = l(\approx \frac{\lambda}{2})$. When we chose $M = 4$ and $N = 4$ it is possible to write the array manifold sufficiently accurate as a function of a limited number of phase modes because $M \ll kR$ and $N \ll k\sqrt{R^2 + z_{max}^2}$. Nine elements in the array are sufficient ($Z > 2M$) to sample the array manifold and to decompose the beamspace manifold vector into an elevation part and an azimuthal part. To demonstrate the combined algorithm UCA-RARE + MUSIC, consider three uncorrelated sources emitting 10000 bit pseudo-random bit sequences. The signals are received in the presence of additive white Gaussian noise. In Table 1 we calculate the mean and standard deviation for an ensemble consisting of 500 im-

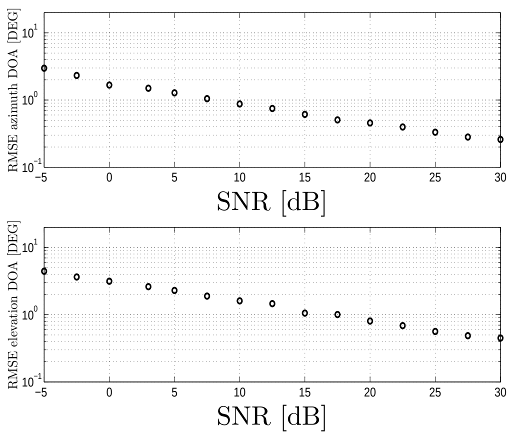

Figure 2: RMSE's of azimuth and elevation angles versus SNR.

plementations: for equally strong signals at DOAs $\{\varphi_1 = 50°, \theta_1 = 50°\}$, $\{\varphi_2 = 135°, \theta_2 = 145°\}$ and $\{\varphi_3 = 280°, \theta_3 = 90°\}$ at different signal-to-noise-ratio (SNR) levels. In figure 2 the Root-mean squared error (RMSE) of the azimuth and elevation DOA estimates are plotted versus the SNR. It is clear that the combined algorithm provides good estimates of the DOAs.

A special case is the situation where two plane waves impinge at the same azimuth angle e.g. two sources impinge at $\{\varphi_1 = 50°, \theta = 45°\}$ and $\{\varphi_1 = 50°, \theta = 80°\}$ at SNR level 30dB. UCA-RARE provides the estimation for the azimuth angle; in figure 3 the MUSIC spectrum is shown as a function of elevation angle, at $\widehat{\varphi} = 50.0143°$. It is clear that two peaks can be distinguished at $\theta = 48°$ and $\theta = 79°$. The symmetry of the antenna array implies that it isn't possible to distinguish between directions θ and $\pi - \theta$ (four peaks are actually observed). The antenna elements are dipole antennas and they can not detect plane waves which impinge at the poles. This property is expressesd by peaks in the MUSIC function at $\theta = 0, \pi$.

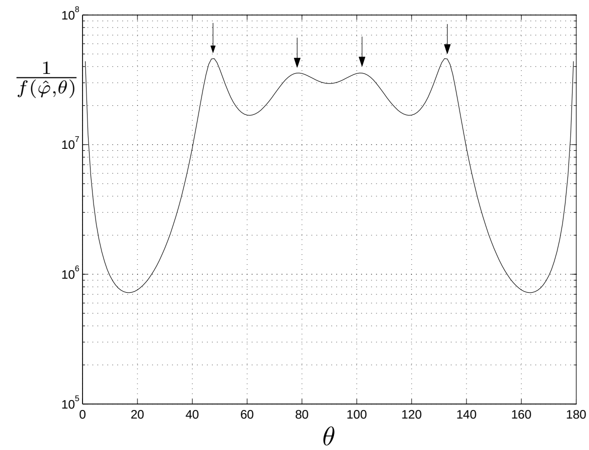

Figure 3: The inverse MUSIC spectrum versus the elevation angle.

4 CONCLUSIONS

A new method for combined azimuth and elevation DOA estimation in UCAs is presented. In a first step an estimation for the azimuth angles is performed by UCA-RARE which decouples the estimation of azimuth angles from the estimation of elevation angles. The estimated azimuth angles enable us to perform a 1D search over the MUSIC spectrum and in order to determine the elevation angles. Innovating in the combination of these two methods UCA-RARE + MUSIC is the fact that mutual coupling is fully taken into account by means of a limited number of parameters, given the expansion of the open-circuit voltages into spherical modes.

Acknowledgments

H. Rogier is a Postdoctoral Research Fellow of the FWO-V. His research was supported by a grant of the DWTC/SSTC, MOTION project and by a travel grant of the FWO-V.

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Session 25: ANTENNAS, SYSTEMS AND ARRAYS

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9th International Conference on Electromagnetics in Advanced Applications ICEAA '05

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September 12-16, 2005

Torino, Italy

ISBN 88-8202-093-2